Certificate

Department of : Mathematics.

Class: 1st BSC MPCS Register No.: 221107102001

Certified that this is the bonafide record of practical work done in the laboratory by the candidate

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during the year 2022-2023

No. of Practicals conducted 05 No. of Practicals attended 05

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Examiners

Index ...

UNIT - I :- DIFFERENTIAL EQUATIONS OF FIRST ORDER AND FIRST DEGREE

Expt. No. 01. Name Date.

1. $(x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy = 0.$

Sol: G.T $(x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy = 0 \longrightarrow ①$

Eq ① comparing with

$M\,dx + N\,dy = 0$

$M = x^2y - 2xy^2$ $\qquad N = -(x^3 - 3x^2y)$

$\frac{\partial M}{\partial y} = x^2 - 2.2xy$ $\qquad \frac{\partial N}{\partial x} = -3x^2 + 3x^2$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

Eq ① Non - Exact Differential Eqn & Homogeneous D.E.

Integration factors $= \frac{1}{Mx + Ny}$

$Mx + Ny = (x^2y - 2xy^2)x + (-x^3 + 3x^2y)y.$

$= \cancel{x^3y} - 2x^2y^2 - \cancel{x^3y} + 3x^2y^2$

$I.F = \frac{1}{3x^2y^2 - 2x^2y^2}$

$= \frac{1}{x^2/y^2}$

Eq ① multiply with I.F

$\left[\frac{x^2y - 2xy^2}{x^2y^2}\right]dx + \left[\frac{-x^3 + 3x^2y}{x^2y^2}\right]dy = 0.$

$= \left(\frac{x^2y}{x^2y^2} - \frac{2xy^2}{x^2y^2}\right)dx + \left(\frac{-x^3}{x^2y^2} + \frac{3x^2y}{x^2y^2}\right)dy = 0$

$\left(\frac{1}{y} - \frac{2}{x}\right)dx + \left(\frac{-x}{y^2} + \frac{3}{y}\right)dy = 0 \rightarrow ②$

Again eq ② compare with $M_1dx + N_1dy = 0.$

$M_1 = \frac{1}{y} - \frac{2}{x}$ $\qquad N_1 = \frac{-x}{y^2} + \frac{3}{y}$

$\frac{\partial M_1}{\partial y} = \frac{-1}{y^2}$ $\qquad \frac{\partial N_1}{\partial x} = -1/y^2$

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$\therefore \frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$

Eq ② is E.D.E

$\therefore u + \int \Phi_y(y) + dy = c \rightarrow$ ③ is a general solution of eq ②

$u_1 = \int M dx = \int \left(\frac{1}{y} - \frac{2}{x}\right) dx$

$= \frac{1}{y} \int 1 dx - 2\int \frac{1}{x} dx$

$= \frac{x}{y} - 2\log x$

$\phi(y)$ = Terms of N not containing x

$= 3/y.$

from ③

$u_1 + \int \phi_1(y) dy = c$

$x/y - 2\log x + 3\int \frac{1}{y} dy = c.$

$\Longrightarrow \boxed{x/y - 2\log x + 3\log y = c}.$

2) $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x) dy = 0.$

Sol: Given that $(y^4+2y)dx + (xy^3+2y^4-4x)dy = 0.$

$M = y^4 + 2y$ $\quad$ $N = xy^3 + 2y^4 - 4x$

$\frac{\partial M}{\partial y} = 4y^3 + 2$ $\quad$ $\frac{\partial N}{\partial x} = y^3 + 0 - 4 = y^3 - 4$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

eq ① is NEDE, NHDE & $T(M) < T(N)$

$\text{I.F} = \int f(x)\, dy \qquad \int e^{K dy}$

(or)

$f(y) = \left[\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M}\right]$

$= \frac{y^3 - 4 - 4y^3 - 2}{y^4 + 2y}$

$= \frac{-3y^3 - 6}{y^4 + 2} = \frac{-3(y^3+2)}{y(y^3+2)} = -3/y.$

$e^{\int -3/y \, dy} = e^{-3\int 1/y \, dy} = e^{-3\log y}$

$= e^{-\log(1/y^3)}$

$= 1/y^3$

eq ① multiply with I.F $= 1/y^3$

$1/y^3 (y^4+2y)dx + 1/y^3 (xy^3+2y^4-4x)dy = 0.$

$(y + 2/y^2)dx + (x+2y-4x/y^3)dy = 0$

compare eq② with $M_1dx + N_1dy = 0.$

$M_1 = y + 2/y^2$ $\qquad$ $N_1 = x+2y - 4x/y^3$

$\frac{\partial M_1}{\partial y} = 1+2(-2y^{-3})$ $\qquad$ $\frac{\partial N_1}{\partial x} = 1+0 - \frac{4}{y^3}$

$= 1 - \frac{4}{y^3}$ $\qquad$ $1 - \frac{4}{y^3}$

$\therefore \frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$

$\therefore$ eq ② is EDE.

$u + \int \phi y \, dy = c \rightarrow$ ③ is the general solution of ②

$u = \int M_1 dx$

$= \int y + 2/y^2 dx = y\int 1dx + 2/y^2 \int 1dx$

$= xy + 2x/y^2$

ϕy = No of terms of N_1 not contains x

$\phi y = 2y$

$\int \phi y \, dy = 2\int y \, dy$

eq ③ $\Rightarrow xy + 2x/y^2 + y^2 = C.$

3) $x^2 y dx - (x^3+y^3)dy = 0.$

Sol: Given that $(x^2 y)dx - (x^3+y^3)dy = 0 \longrightarrow$ ①

Eq ① compare with $Mdx + Ndy = 0.$

$M = x^2y$ $\qquad$ $N = -x^3 - y^3$

$\frac{\partial M}{\partial y} = x^2$ $\qquad$ $\frac{\partial N}{\partial x} = -3x^2$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

Eq ① is NEDE

from ① $x^2y\,dx = (x^3+y^3)dy$

$$\frac{dy}{dx} = \frac{x^2y}{x^3+y^3}$$

let $f(x,y) = \frac{x^2y}{x^3+y^3}$

Put $x = kx$, $y = ky$.

$$f(kx,ky) = \frac{(kx)^2(ky)}{(kx)^3+(ky)^3}$$

$$= k^0\frac{x^2y}{x^3+y^3}$$

$$f(kx,ky) = k^0 f(x,y)$$

H.D.E of eq ① is NEDE & NHDE

Degree of $k=0$.

$$I.F = \frac{1}{Mx+Ny}$$

$$Mx+Ny = x^2y(x) + (-x^3-y^3)y$$

$$= x^3y - x^3y - y^4$$

$$= Mx+Ny = -y^4$$

$$I.F = \frac{1}{Mx+Ny} = -\frac{1}{y^4}$$

eq ① multiply with $I.F = -\frac{1}{y^4}$

$$x^2y\,dx - [x^3+y^3]\,dy = 0$$

$$\left(-\frac{x^2y}{y^4}\right)dx + \left(\frac{x^3}{y^4}+\frac{y^3}{y^4}\right)dy = 0$$

$$-\frac{x^2}{y^3}dx + \left(\frac{x^3}{y^4}+\frac{1}{y}\right)dy = 0 \rightarrow ②$$

Again eq ② compare with $M_1dx + N_1dy = 0$.

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$M_1 = \frac{x^2}{y^3}$ $\quad N_1 = \frac{x^3}{y^4} + \frac{1}{y}$

$\frac{\partial M_1}{\partial y} = -x^2$ $\quad \frac{\partial N_1}{\partial x} = \frac{3x^2}{y^4}$

$\frac{\partial M_1}{\partial y} = \frac{3x^2}{y^4}$ $\quad \frac{\partial N_1}{\partial x} = \frac{3x^2}{y^4}$

$\therefore \frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$

Eq (2) is an E.D.E.

$u + \int \phi(y)\,dy = c$ is a general solution of eq (3)

$u_1 = \int M_1 dx = \int \frac{-x^2}{y^3} dx$

$-1/y^3 \int x^2 dx$

$= -1/y^3 \left(\frac{x^{2+1}}{2+1} \right)$

$= \frac{-x^3}{3y^3}$

$\phi(y)$ = terms of N_1 not contains x.

$= 1/y$

from (3)

$-x^3/3y^3 + \int 1/y \, dy = c$

$\boxed{= \frac{-x^3}{3y^3} + \log y = c}$.

Certificate

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Examiners

UNIT - I

Expt. No. Name Date. Page No. 1

DIFFERENTIAL EQUATIONS OF FIRST ORDER AND FIRST DEGREE

1. $[1+e^{x/y}]dx + [1-x/y]e^{x/y}dy = 0$

Sol: Given that

$$1+e^{x/y}dx + [1-x/y]e^{x/y}dy = 0 \quad \text{—} ①$$

Eqn compare with $Mdx + Ndy = 0$

$M = 1+e^{x/y}$ $\qquad N = (1-x/y)e^{x/y}$

$$\frac{\partial M}{\partial y} = 0 + e^{x/y}[x(1-1/y^2)]$$

$$= \frac{-xe^{x/y}}{y^2}$$

$$N = (1-x/y)e^{x/y} = e^{x/y} - (x/y)e^{x/y}$$

$$\frac{\partial N}{\partial x} = e^{x/y}\left(\frac{1}{y}\right) - \left[\frac{x}{y}\cdot e^{x/y}\frac{1}{y} + e^{x/y}\cdot\frac{1}{y}\right]$$

$$e^{x/y}\frac{1}{y} - x\frac{e^{x/y}}{y^2} - e^{x/y}\cdot\frac{1}{y}$$

$$\frac{-xe^{x/y}}{y^2}$$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Given eqn 1 is E.D.E

$\therefore u + \int \phi y \, dy = c \rightarrow c$ is a General solution of eqn

$$u = \int_{(y)} M dx$$

ϕy = Terms of 'N' not containing 'x'

$$u = \int M dx = \int 1 + e^{x/y}dx$$

$$= \int 1 dx + \int e^{x/y}dx$$

$$x + \frac{e^{x/y}}{(1/y)}$$

$$u = x + y e^{x/y}$$

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$\therefore y = 0$ from 2

$$x + ye^{x/y} + \int 0\, dy = c$$

$$x + ye^{x/y} = c$$

2 $x^2 y\, dx - (x^3 + y^3)\, dy = 0$

Sol Given that

$$x^2 y\, dx - (x^3 + y^3)\, dy = 0 \quad \text{---} ①.$$

Given eqn1 compare with $Mdx + Ndy = 0$

$M = x^2 y$ $\qquad$ $N = -x^3 - y^3$

$\frac{\partial M}{\partial y} = x^2$ $\qquad$ $\frac{\partial N}{\partial x} = -3x^2$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

eqn1 is NEDE from 1

$x^2 y\, dy = (x^3 + y^3)\, dy$

$$\frac{dy}{dx} = \frac{x^2 y}{x^3 + y^3}$$

$$\text{let } f(x, y) = \frac{x^2 y}{x^3 + y^3}$$

put $x = kx$, Put $y = ky$.

$$f(kx, ky) = \frac{(kx)^2 (ky)}{(kx)^3 + (ky)^3}$$

$$k^0 \frac{x^2 y}{x^3 + y^3}$$

$f(kx, ky) = k^0 f(x,y)$

H.D.E eqn 1 is N.E.D.E & H.D.E degree of k = 0

$$I.F = \frac{1}{Mx+Ny}$$

$$Mx+Ny = x^2y(x) + (-x^3 - y^3)y$$
$$= \cancel{x^3y} - \cancel{x^3y} - y^4$$
$$= Mx+Ny = y^4$$
$$I.F = \frac{1}{Mx+Ny} = -\frac{1}{y^4}$$

eqn 1 multiplying with $I.F = \frac{-1}{y^4}$

$$x^2y\,dx - [x^3 + y^3]\,dy = 0$$
$$\left(-\frac{x^2}{y^4}y\right)dx + \left(\frac{x^3}{y^4} + \frac{y^3}{y^4}\right)dy = 0$$
$$-\frac{x^2}{y^3}dx + \left(\frac{x^3}{y^4} + \frac{1}{y}\right)dy = 0 \rightarrow 2$$

Again eqn 2 compare with

$$M_1dx + N_1dy = 0$$

$$M_1 = \frac{x^2}{y^3} \qquad N_1 = \frac{x^3}{y^4} + \frac{1}{y}$$

$$\frac{\partial M_1}{\partial y} = -x^2 \qquad N_1 = \frac{3x^2}{y^4}$$

$$\frac{\partial M_1}{\partial y} = \frac{3x^2}{y^4} \qquad \frac{\partial N_1}{\partial x} = \frac{3x^2}{y^4}$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N}{\partial x}.$$

Eqn 2 is an E.D.E

$u_1 + \int \phi_1(y)dy = c$ is a General sol of eqn -

$$u_1 = \int m_1\, dx = \int \frac{-x^2}{y^3}\, dx$$

$$\frac{-1}{y^3} \int x^2 dx$$

$$\frac{-1}{y^3} \left[\frac{x^{2+1}}{2+1} \right]$$

$$= \frac{-x^3}{3y^3}$$

$\phi_1(y)$ = Term of N_1 not containing $x = 1/y$ from 3

$$\frac{-x^3}{3y^3} + \int \frac{1}{y}\, dy = c$$

$$\frac{-x^3}{3y^3} + \log y = c.$$

3. $(x^2y - 2xy^2)\, dx - (x^3 - 3x^2y)\, dy = 0$

Sol: $(x^2y - 2xy^2)\, dx - (x^3 - 3x^2y)dy = 0 \rightarrow 1$

Eqn 1 compare with

$$Mdx + Ndy = 0$$

$M = x^2y - 2xy^2$ $\qquad$ $N = -(x^3 - 3x^2y)$

$\frac{\partial M}{\partial y} = x^2 - 2.x\,2y$ $\qquad$ $\frac{\partial N}{\partial x} = -3x^2 + 3x^2$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

Eqn 1 Non-Exact Differential equation & Homogeneous Differential equation.

Integration factor.

$$\frac{1}{Mx+Ny}$$

$$Mx+Ny = (x^2y-2xy^2)x+(-x^3+3x^2y)y.$$
$$= \cancel{x^3y}-2x^2y^2-\cancel{x^3y}+3x^2y^2$$

$$I.F = \frac{1}{3x^2y^2-2x^2y^2} = \frac{1}{x^2y^2(3-2)}$$

$$\frac{1}{x^2y^2}$$

eqn1 multiply with I.F

$$\left[\frac{x^2y-2xy^2}{x^2y^2}\right]dx + \left[\frac{-x^3+3x^2y}{x^2y^2}\right]dy=0$$

$$\left[\frac{\cancel{x^2y}}{\cancel{x^2y^2}} - \frac{2\cancel{xy^2}}{\cancel{x^2y^2}}\right]dx + \left[\frac{-x^3}{x^2y^2}+\frac{3x^2y}{x^2y^2}\right]dy=0$$

$$\left[\frac{1}{y}-\frac{2}{x}\right]dx + \left[\frac{x}{y^2}+\frac{3}{y}\right]dy=0 \rightarrow 2$$

Again eqn2 compare with

$$M_1dx + N_1dy = 0$$

$$M_1 = 1/y - \frac{2}{x} \qquad N = \frac{-x}{y^2}+\frac{3}{y}$$

$$\frac{\partial M_1}{\partial y} = -\frac{1}{y^2} \qquad \frac{\partial N_1}{\partial x} = -1/y^2$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial y}$$

eqn2 is E.D.E

$u_1 + \int \phi(y)dy = c \rightarrow 3$ is a General solution of eqn2

$$= \frac{y^3 - 4 - 4y^3 - 2}{y^4 + 2y}$$

$$= \frac{-3y^3 - 6}{y^4 + 2} = \frac{-3(y^3 + 2)}{y(y^3 + 2)} = -3/y$$

$$e^{\int -3/y\,dy} = e^{-3\int 1/y\,dy} = e^{-3\log y}$$

$$= e^{\log(1/y^3)}$$

$$= 1/y^3$$

eqn 1 multiply with I.F $= 1/y^3$

$$1/y^3 (y^4 + 2y)dx + 1/y^3 (xy^3 + 2y^4 - 4x)dy = 0$$

$$\left(y + \frac{2}{y^2}\right)dx + (x + 2y - 4x/y^3)dy = 0.$$

compare eqn 2 with $M_1 dx + N_1 dy = 0$

$M_1 = y + 2/y^2$ $\quad N_1 = x + 2y - 4x/y^3$

$\frac{\partial M_1}{\partial y} = 1 + 2(-2y^{-3})$ $\quad \frac{\partial N_1}{\partial x} = 1 + 0 - 4/y^3$

$= 1 - \frac{4}{y^3}$ $\quad 1 - \frac{4}{y^3}$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

$\therefore$ eqn 2 is E.D.E

$u + \int \phi y \, dy = c \rightarrow 3$ is a General Solution of eqn 2

$u = \int M_1 dx$

$\int (y + \frac{2}{y^2}) dx = y \int 1 dx + \frac{2}{y^2} \int 1 dx$

$xy + \frac{2x}{y^2}$

ϕy = NO of terms of N_1 not contains x

$\phi y = 2y$

$\phi y \, dy = 2 \int y \, dy$

eqn 3 $\Rightarrow xy + 2x/y^2 + y^2 = c$

Certificate

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Examiners

Index ...

UNIT-I :- DIFFERENTIAL EQUATIONS OF FIRST ORDER
B1 FIRST DEGREE

Expt. No. 01 Name Date. Page No. 1

1. $(x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy = 0$

A) $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0 \rightarrow (1)$

Eqn 1 comparing with

$Mdx + Ndy = 0$

$M = x^2y - 2xy^2 \qquad N = -(x^3 - 3x^2y)$

$\qquad\qquad\qquad\qquad = -x^3 + 3x^2y$

$\frac{\partial M}{\partial y} = x^2 - 2\cdot x\,2y \qquad \frac{\partial N}{\partial x} = -3x^2 + 3x^2$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

Eqn 1 Non-exact Differential Eqn & Homogeneous D.E

Integration factors $= \frac{1}{Mx + Ny}$

$Mx + Ny = (x^2y - 2xy^2)x + (-x^3 + 3x^2y)y$

$= x^3y - 2x^2y^2 - x^3y + 3x^2y^2$

$If = \frac{1}{3x^2y^2 - 2x^2y^2} = \frac{1}{x^2y^2(3-2)}$

$= \frac{1}{x^2y^2}$

Eqn 1 multiply with I.f

$\left[\frac{x^2y - 2xy^2}{x^2y^2}\right]dx + \left[\frac{-x^3 + 3x^2y}{x^2y^2}\right]dy = 0$

$\left[\frac{x^2y}{x^2y^2} - \frac{2xy^2}{x^2y^2}\right]dx + \left[\frac{-x^3}{x^2y^2} + \frac{3x^2y}{x^2y^2}\right]dy = 0$

$\left[\frac{1}{y} - \frac{2}{x}\right]dx + \left[-\frac{x}{y^2} + \frac{3}{y}\right]dy = 0 \rightarrow (2)$

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Again eqn 2 compare with $M_1 dx + N_1 dy = 0$

$M_1 = \frac{1}{y} - \frac{2}{x}$ $\quad N_1 = -\frac{x}{y^2} + 3/y$

$\frac{\partial M_1}{\partial y} = -1/y^2$ $\quad \frac{\partial N_1}{\partial x} = -1/y^2$

$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial y}$

Eqn 2 is E.D.E

$\therefore u_1 + \int \phi_1(y)\,dy = C \rightarrow 3$ is a

G.S of eqn 2

$u_1 = \int M_1 dx = \int \left(1/y - \frac{2}{x}\right) dx$

$= 1/y \int 1 dx - 2\int 1/x\, dx$

$= x/y - 2\log x$

$\phi_1(y) =$ Terms of N not containing $x = 3/y$

from ③

$u_1 + \int \phi_1(y)\,dy = C$

$x/y - 2\log x + 3\int 1/y\, dy = C$

$\boxed{x/y - 2\log x + 3\log y = C}$

② $(y^4 + 2y)\,dx + (xy^3 + 2y^4 - 4x)\,dy = 0$

A) Given that $(y^4 + 2y)\,dx + (xy^3 + 2y^4 - 4x)\,dy = 0$

eqn compare with $M dx + N dy = 0$

$M = y^4 + 2y$ $\quad N = xy^3 + 2y^4 - 4x$

$\frac{\partial M}{\partial y} = 4y^3 + 2$ $\quad \frac{\partial N}{\partial x} = y^3 + 0 - 4 = y^3 - 4$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

eqn 1 is NEDE, NHDE & $T(m) < T(N)$

$I.F = \int f(x)\,dy \qquad \int f\,dy.$

(or)

$$f(y) = \left[\frac{\frac{\partial N}{\partial x} - \frac{\partial m}{\partial y}}{m}\right]$$

$$= \frac{y^3 - y - 4y^3 - 2}{y^4 + 2y}$$

$$= \frac{-3y^3 - 6}{y^4 + 2} = \frac{-3(y^3 + 2)}{y(y^3 + 2)} = \frac{-3}{y}$$

$$e^{\int -3/y\,dy} = e^{-3\int 1/y\,dy} = e^{-3\log y}$$

$$= e^{\log(1/y^3)}$$

$$= 1/y^3$$

eqn 1 multiply with $I.f = 1/y^3 (xy^3 + 2y^4 - 4x)dy = 0$

$1/y^3 (y^4 + 2y)\,dx + 1/y^3 (xy^3 + 2y^4 - 4x)dy = 0$

$(y + 2/y^2)\,dx + (x + 2y - 4x/y^3)\,dy = 0$

Compare eqn 2 with $m_1 dx + N_1 dy = 0$

$m_1 = y + 2/y^2 \qquad N_1 = x + 2y - 4x/y^3$

$\frac{\partial m_1}{\partial y} = 1 + 2(-2y^{-3}) \qquad \frac{\partial N_1}{\partial x} = 1 + 0 - 4/y^3$

$= 1 - 4/y^3 \qquad 1 - 4/y^3$

$\therefore$ eqn 2 is E.D.E

$u + \int \phi_y\,dy = c \rightarrow 3.$ G.S of eqn 2

$u = \int m_1 dx$

$= \int (y + 2/y^2)dx = y\int 1dx + 2/y^2 \int 1dx$

$= xy + 2x/y^2$

ϕ_y = No. of terms of N_1 Not Contains x

$\phi_y = 2y$

$\int 2y\,dy = 2\int y\,dy$

eqn 3 $\Rightarrow xy + 2x/y^2 + y^2 = C$

3) $x^2 y\,dx - (x^3+y^3)\,dy = 0$

A) G.t $x^2 y\,dx - (x^3+y^3)\,dy = 0$ — ①

Given eqn ① compare with

$M\,dx + N\,dy = 0$

$M = x^2 y$ $\quad N = -x^3 - y^3$

$\frac{\partial M}{\partial y} = x^2$ $\quad \frac{\partial N}{\partial x} = -3x^2$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

eqn ① is N.E.D.E

from ① $x^2 y\,dx = (x^3+y^3)\,dy$

$\frac{dy}{dx} = \frac{x^2 y}{x^3+y^3}$

let $f(x,y) = \frac{x^2 y}{x^3+y^3}$

Put $x = kx$, $y = ky$

$f(kx,ky) = \frac{(kx)^2(ky)}{(kx)^3+(ky)^3}$

$= k^0 \frac{x^2 y}{x^3+y^3}$

$f(kx,ky) = k^0 f(x,y)$

H.D.E eqn ① is N.E.D.E & H.D.E

degree of $k = 0$

$I.F = \frac{1}{Mx+Ny}$

$Mx + Ny = x^2y(x) + (-x^3 - y^3)y$

$= x^3y - x^3y - y^4$

$= Mx + Ny = -y^4$

$I.F = \frac{1}{Mx+Ny} = -1/y^4$

eqn 1 multiplying with $I.F = -1/y^4$

$x^2y\,dx - [x^3+y^3]\,dy = 0$

$\left[\frac{-x^2y}{y^4}\right]dx + \left[\frac{x^3}{y^4} + \frac{y^3}{y^4}\right]dy = 0$

$\frac{-x^2}{y^3}\,dx + \left[\frac{x^3}{y^4} + 1/y\right]dy = 0 \rightarrow (2)$

Again eqn 2 compare with

$M_1dx + N_1dy = 0$

$M_1 = \frac{x^2}{y^3}$ $N_1 = \frac{x^3}{y^4} + 1/y$

$\frac{\partial M_1}{\partial y} = -x^2$ $N_1 = \frac{3x^2}{y^4}$

$\frac{\partial M_1}{\partial y} = \frac{3x^2}{y^4}$ $\frac{\partial N_1}{\partial x} = \frac{3x^2}{y^4}$

$\frac{\partial M_1}{\partial y} = \frac{\partial N}{\partial x}$

Eqn 2 is an E.D.E

$u_1 + \int \phi_1(y)\,dy = C$ is a general sol of eqn 3

$u_1 = \int M_1\,dx = \int -x^2/y^3\,dx$.

$= -1/y^3 \int x^2\,dx$

$= -1/y^3\left(\frac{x^{2+1}}{2+1}\right)$

$= \frac{-x^3}{3y^3}$

$\phi_1(y)$ = Term of N, Not
$= 1/y$

from ③

$\frac{-x^3}{3y^3} + \int 1/y \, dy = c$

$$\boxed{\frac{-x^3}{3y^3} + \log y = c}$$

④ $x\,dy/dx + y = y^2 \log x$

G.t $x\,dy/dx + y = y^2 \log x$

$\frac{dy}{dx} + y/x = \frac{y^2 \log x}{x} \rightarrow ①$

eqn 1 is BNE in y

$P(x) = 1/x$, $Q(x) = \frac{\log x}{x}$, $n = 2$

eqn 1 divided with y^2

$1/y^2 \cdot dy/dx + 1/x \cdot 1/y = \log x/x$

$y^{-2} \frac{dy}{dx} + \frac{1}{x} y^{-1} = \frac{\log x}{x}$ — ②

$y^{-1} = t$

$-1 \cdot y^{-1-1} \frac{dy}{dx} = \frac{dt}{dx}$

$y^{-2} \frac{dy}{dx} = -\frac{dt}{dx} \rightarrow ③$

from ② & ③

$-\frac{dt}{dx} + 1/x \cdot t = \frac{\log x}{x}$

$$\frac{dt}{dx} + \left(-\frac{1}{x}\right)\cdot t = \frac{-\log x}{x} \quad \text{—(4)}$$

eqn 4 is L.D.E in 't'

$\therefore t\,[I.f] = \int Q(x)\,(I.f)\,dx + C$ is a G.S of eqn 4

$I.f = e^{\int P(x)\,dx}$

$= e^{\int -1/x\,dx}$

$= e^{-\log x}$

$= e^{\log x^{-1}}$

$= x^{-1} = 1/x$

$I.f = 1/x$

from 4

$t\,\frac{1}{x} = \int -\frac{\log x}{x}\cdot\frac{1}{x}\,dx + C = -\int x^{-2}\log x\,dx + C$

$= -\int x^{-2}\log x\,dx + C = -\left[\log x \int x^{-2}dx - \int\left(\frac{1}{x}\cdot\int x^{-2}dx\right)dx\right] + C$

$= -\left[\log x\left[\frac{x^{-2+1}}{-2+1}\right] - \int \frac{1}{x}\left[\frac{x^{-2+1}}{-2+1}\,dx\right]\right] + C$

$= \left[\frac{\log x}{x} - \int x^{-2}\,dx\right] + C$

$t\,\frac{1}{x} = \frac{\log x}{x} + \frac{1}{x} + C$

$y^{-1}\cdot\frac{1}{x} = \frac{\log x + 1}{x} + C$

$\frac{1}{xy} = \frac{\log x + 1}{x} + C$

Certificate

Department of : MATHEMATICS

Class: Ist BSC [MPCs] Register No.: 221107102004

Certified that this is the bonafide record of practical work done in the laboratory by the candidate Kurivella Vyshnavi during the year 2022 - 2023

No. of Practicals conducted 05 No. of Practicals attended 05

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1. M. Akhil

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Examiners

Index •••

DIFFERENTIAL EQUATIONS

UNIT-I :- Differential equations of 1^{st} order & 1^{st} degree

* SOLVED PROBLEMS *

1. $(x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy = 0$

Sol: Given that $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$

eqn ① compare with $Mdx + Ndy = 0$

$M = x^2y - 2xy^2$ $\qquad N = -(x^3 - 3x^2y)$

$\qquad\qquad\qquad\qquad = -x^3 + 3x^2y$

$\frac{\partial M}{\partial y} = x^2 - 2xy.2$ $\qquad \frac{\partial N}{\partial x} = -3x^2 + 3x^2y$

$= x^2 - 4xy$ $\qquad\qquad = -3x^2 + 6xy$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$\therefore$ eqn ① Non - Exact Differential equation & Homogeneous D.E.

$$\text{Integrating Factors} = \frac{1}{Mx + Ny}$$

$Mx + Ny = (x^2y - 2xy^2)x + (-x^3 + 3x^2y)y$

$= \cancel{x^3y} - 2x^2y^2 - \cancel{x^3y} + 3x^2y^2$

$\text{I.F} = \frac{1}{3x^2y^2 - 2x^2y^2} = \frac{1}{x^2y^2(3-2)}$

$= \frac{1}{x^2y^2}$

eqn ① multiply with I.F.

$$\left(\frac{x^2y - 2xy^2}{x^2y^2}\right)dx + \left(\frac{-x^3 + 3x^2y}{x^2y^2}\right)dy = 0$$

$$\left(\frac{x^2y}{x^2y^2} - \frac{2xy^2}{x^2y^2}\right)dx + \left(\frac{-x^3}{x^2y^2} + \frac{3x^2y}{x^2y^2}\right)dy = 0$$

$$\left(\frac{1}{y}-\frac{2}{x}\right)dx+\left(\frac{-x}{y^2}+\frac{3}{y}\right)dy=0 \longrightarrow (2)$$

Again eqn (2) compare with $M_1dx + N_1dy = 0$

$M_1 = \frac{1}{y} - \frac{2}{x}$ | $N_1 = -\frac{x}{y^2} + \frac{3}{y}$

$\frac{\partial M_1}{\partial y} = -1/y^2$ | $\frac{\partial N}{\partial x} = \frac{-1}{y^2}$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

eqn (2) is Exact Differential Equation

$\therefore$ $u_1 + \int \phi_{1(y)} dy = c \rightarrow (3)$ is a General Solution of eqn (2)

$$u_1 = \int M_1 dx = \int\left(\frac{1}{y}-\frac{2}{x}\right)dx$$
$$= 1/y \int 1 dx - 2\int 1/x \, dx$$
$$= x/y - 2\log x$$

$\phi_{(y)}$ = Terms of 'N_1' not containing 'x'
$= 3/y$

from (3)

$$u_1 + \int \phi_{1(y)} dy = c$$

$$x/y - 2\log x + 3\int 1/y \, dy = c.$$

$$\boxed{x/y - 2\log x + 3\log c = c}$$

2. $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0.$

Soln: Given that $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0 \rightarrow (1)$

eqn(1) compare with $Mdx + Ndy = 0$

$M = y^4 + 2y \quad | \quad N = xy^3 + 2y^4 - 4x$

$\frac{\partial M}{\partial y} = 4y^3 + 2 \quad | \quad \frac{\partial N}{\partial x} = y^3 + 0 - 4 = y^3 - 4.$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}.$$

eqn(1) is NEDE, NHDE and $T(M) < T(N)$

$$\text{I.F.} = e^{\int f(x)dx} \text{ (or) } e^{\int k dy}$$

$$f(y) = \left[\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M}\right]$$

$$= \frac{y^3 - 4 - 4y^3 - 2}{y^4 + 2y}$$

$$= \frac{-3y^3 - 6}{y^4 + 2} = \frac{-3(y^3+2)}{y(y^3+2)} = \frac{-3}{y}$$

$$e^{\int -3/y\, dy} = e^{-3\int \frac{1}{y} dy} = e^{-3\log y}$$

$$= e^{\log(1/y^3)}$$

$$= 1/y^3$$

eqn(1) multiply with I.F. $= 1/y^3$

$$\frac{1}{y^3}(y^4 + 2y)dx + \frac{1}{y^3}(xy^3 + 2y^4 - 4x)dy = 0$$

$$(y + 2/y^2)dx + \left(x + 2y - \frac{4x}{y^3}\right)dy = 0 \quad (2)$$

eqn(2) compare with $M_1 dx + N_1 dy = 0$

$M_1 = y + 2/y^2$

$\frac{\partial M_1}{\partial y} = 1 + 2(-2y^{-3})$

$= 1 - 4/y^3$

$N_1 = x + 2y - 4x/y^3$

$\frac{\partial N_1}{\partial x} = 1 + 0 - 4/y^3$

$= 1 - 4/y^3$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

$\therefore$ eqn ② is Exact Differential Equation.

$\mu_1 + \int \phi_1(y)\, dy = c \rightarrow$ ③ is the General soln of eqn ②

$\mu_1 = \int M_1 dx$

$= \int \left(y + \frac{2}{y^2}\right) dx = y \int 1 dx + \frac{2}{y^2} \int 1 dx.$

$= xy + 2x/y^2$

$\phi_1(y) dy$ = No. of terms of N_1 not contains 'x'

$\phi_1(y) = 2y$

$\phi_1(y) dy = 2\int y\, dy = \frac{\cancel{2} y^2}{\cancel{2}}$

$= y^2$

from ③

$xy + 2x/y^2 + y^2 = c$

3. Solve $x^2 y dx - (x^3 + y^3) dy = 0$

Soln: Given that $x^2 y dx - (x^3 + y^3) dy = 0 \rightarrow$ ①

$x^2 \cdot y dx = (x^3 + y^3) dy$

$\frac{dy}{dx} = \frac{x^2 y}{x^3 + y^3} \rightarrow$ ②

$f(x, y) = \frac{x^2 y}{x^3 + y^3}$

$f(kx + ky) = \frac{(kx)^2 \cdot (ky)}{(kx)^3 + (ky)^3}$

$$= \frac{\cancel{k^3}(x^2 \cdot y)}{\cancel{k^3}(x^3+y^3)}$$

$$= k^0 \left[\frac{x^2 \cdot y}{x^3+y^3}\right]$$

$$= k^0 \left[\frac{x^2 \cdot y}{x^3+y^3}\right]$$

clearly eqn ① is H.D.E.

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \longrightarrow ③$$

from eqn ② & ③

$$v + x\frac{dv}{dx} = \frac{x^2 \cdot vx}{x^3+(vx)^3}$$

$$= \frac{\cancel{x^3} v}{(1+v^3)\cancel{x^3}}$$

$$= \frac{v}{1+v^3}$$

$$v + x\frac{dv}{dx} = \frac{v}{1+v^3}$$

$$x\frac{dv}{dx} = \frac{v}{1+v^3} - v$$

$$= \frac{\cancel{v} - \cancel{v} - v^4}{1+v^3}$$

$$x\frac{dv}{dx} = \frac{-v^4}{1+v^3}$$

By Seperating Variable Method

$$\left[\frac{(1+v^3)}{v^4}\right] dv = \frac{-dx}{x}$$

$$(v^{-4} + 1/v)\,dv = -1/x\,dx$$

Integrating on both sides

$$\int v^{-4}dv + \int 1/v\,dv = -\int 1/x\,dx$$

$$\frac{v^{-4+1}}{-4+1} + \log v = -\log x + c$$

$$-1/3\,v^{-3} + \log v + \log x = c.$$

$$-1/3 \cdot \frac{1}{\left(\frac{y}{x}\right)^3} + \log\left(\frac{y}{x}\cdot x\right) = c \qquad \boxed{\begin{array}{l} \because y = vx \\ v = y/x \end{array}}$$

$$-\frac{x^3}{3y^3} + \log y = c.$$

$$\log y - x^3/3y^3 = c$$

4. Solve $[1+e^{x/y}]\,dx + e^{x/y}[1-x/y]\,dy = 0.$

Soln: Given that, $(1+e^{x/y})dx + e^{x/y}(1-x/y)dy = 0 \rightarrow (1)$

eqn (1) compare with $Mdx + Ndy = 0$

$M = 1+e^{x/y}$

$$\frac{\partial M}{\partial y} = 0 + e^{x/y}\left[x\left(-1/y^2\right)\right] = \frac{-xe^{x/y}}{y^2}$$

$N = (1-x/y)e^{x/y} = e^{x/y} - \left[e^{x/y}.x/y\right]$

$$\frac{\partial N}{\partial x} = e^{x/y}(1/y) - \left[1/y.e^{x/y} + e^{x/y}.1/y\right]$$

$$= e^{x/y}.1/y - \frac{xe^{x/y}}{y^2} - e^{x/y}.1/y$$

$$= \frac{-xe^{x/y}}{y^2} \qquad \boxed{\frac{d}{dx}e^{ax} = ae^{ax}}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$$

clearly eqn (1) is Exact Differential Eqn.

$\therefore\ u + \int \phi_y\,dy = c \rightarrow (2)$ is a General soln of eqn (1)

$$u = \int_{(y)} M\,dx$$

ϕ_y = Terms of 'N' not containing 'x'

$$u = \int M\,dx = \int 1 + e^{x/y}\,dx \qquad \boxed{\int e^{ax} = e^{ax}/a}$$

$$= \int 1\,dx + \int e^{x/y}\,dx.$$

$$= x + \frac{e^{x/y}}{(1/y)}$$

$$u = x + ye^{x/y}$$

$$\phi_y = 0$$

from (2)

$$x + ye^{x/y} + \int 0\,dy = c.$$

$$\Rightarrow x + ye^{x/y} = c$$

Certificate

Department of : MATHEMATICS

Class: I^{st} BSc[MPCs] Register No.: 221107102005

Certified that this is the bonafide record of practical work done in the laboratory by the candidate Palaparthi Veera Lakshmi

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Examiners

Index ...

Sr. No.	Name of the Experiment	Page No.	Date of Experiment	Date of Submission	Remarks
	* Differential Equations *				
1	Differential Equation of First Order and first Degree	1-9			
2	Differential Equation of First Order but not of the first degree	10-17			
3	Higher Order Linear Differential Equations - I	18-24			
4	Higher Order Linear Differential Equations - II	25-33			
5	Higher Order Linear Differential Equations - III	34-45			

DIFFERENTIAL EQUATIONS

UNIT-1 Differential Equations of 1^{st} Order & 1^{st} degree

* SOLVED PROBLEMS *

① $(x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy = 0$

Sol:- Given that

$$(x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy = 0 \quad \rightarrow ①$$

Equation ① Compare with $Mdx + Ndy = 0$

$M = x^2y - 2xy^2$

$\frac{\partial M}{\partial y} = x^2 - 2xy2$

$= x^2 - 4xy$

$N = -(x^3 - 3x^2y)$

$= -x^3 + 3x^2y$

$\frac{\partial N}{\partial x} = -3x^2 + 3x^2y$

$= -3x^2 + 6xy$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

Equation ① Non-Exact Differential Equation & Homogeneous Differential Equation.

Integrating factors $= \frac{1}{Mx+Ny}$

$Mx+Ny = (x^2y - 2xy^2)x + (-x^3 + 3x^2y)y$

$= (x^3y - 2x^2y^2 - x^3y + 3x^2y^2)$

$$I.F = \frac{1}{3x^2y^2 - 2x^2y^2} = \frac{1}{x^2y^2(3-2)}$$

$$= \frac{1}{x^2y^2}$$

Equation ① multiply with I.F

$$\left(\frac{x^2y - 2xy^2}{x^2y^2}\right)dx + \left(\frac{-x^3 + 3x^2y}{x^2y^2}\right)dy = 0$$

$$\left(\frac{\cancel{x^2y}}{x^2y^{\cancel{2}}} - \frac{2\cancel{x}\cancel{y^2}}{x^{\cancel{2}}\cancel{y^2}}\right)dx + \left(\frac{-x^{\cancel{3}}}{\cancel{x^2}y^2} + \frac{3\cancel{x^2}\cancel{y}}{\cancel{x^2}y^{\cancel{2}}}\right)dy = 0$$

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$$\left(\frac{1}{y}-\frac{2}{x}\right) dx + \left(\frac{-x}{y^2}+\frac{3}{y}\right) dy = 0 \longrightarrow (2)$$

Again eqn (2) Compare with $M_1dx + N_1dy = 0$

$M_1 = \frac{1}{y} - \frac{2}{x}$ | $N_1 = \frac{-x}{y^2} + \frac{3}{y}$

$\frac{\partial M_1}{\partial y} = \frac{-1}{y^2}$ | $\frac{\partial N_1}{\partial y} = \frac{-1}{y^2}$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

Equation (2) is Exact Differential Equation

$\therefore u_1 + \int \phi_1(y)\, dy = c \longrightarrow (3)$ is a General Solution of eqn (2)

$$u_1 = \int M_1 dx = \int (1/y - 2/x)\, dx$$
$$= 1/y \int 1 dx - 2\int 1/x\, dx$$
$$= x/y - 2\log x$$

$\phi_1(y)$ = Terms of 'N_1' not Containing 'x'
$= 3/y$

from (3)

$$u_1 + \int \phi_1(y)\, dy = c$$

$$x/y - 2\log x + 3\int 1/y\, dy = c$$

$$\boxed{x/y - 2\log x + 3\log c = c}$$

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(2) $(y^4+2y)dx + (xy^3+2y^4-4x)dy = 0$

Sol: Given that $(y^4+2y)dx + (xy^3+2y^4-4x)dy = 0 \longrightarrow (1)$

Equation (1) Compare with $Mdx + Ndy = 0$

$M = y^4+2y$ $\qquad$ $N = xy^3+2y^4-4x$

$\frac{\partial M}{\partial y} = 4y^3+2$ $\qquad$ $\frac{\partial N}{\partial x} = y^3+0-4$

$= y^3-4$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

Equation (1) NEDE, NHDE and T(M) < T(N)

$$I.F = e^{\int f(x)dx} \text{ (or) } e^{\int k dy}$$

$$f(y) = \left[\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M}\right]$$

$$= \frac{y^3-4-4y^3-2}{y^4+2y}$$

$$= \frac{-3y^3-6}{y^4+6} = \frac{-3(y^3+2)}{y(y^3+2)} = \frac{-3}{y}$$

$$e^{\int -3/y\, dy} = e^{-3\int 1/y\, dy} = e^{-3\log y}$$

$$= e^{\log(1/y^3)}$$

$$= 1/y^3$$

Equation (1) multiply with $I.F = 1/y^3$

$$\frac{1}{y^3}(y^4+2y)dx + \frac{1}{y^3}(xy^3+2y^4-4x)dy = 0$$

$$(y+2/y^2)dx + (x+2y-4x/y^3)dy = 0 \longrightarrow (2)$$

Equation (2) Compare with $M_1dx + N_1dy = 0$

$$M_1 = y + 2/y^2$$
$$\frac{\partial M_1}{\partial y} = 1 + 2(-2y^{-3})$$
$$= 1 - 4/y^3$$

$$N_1 = x + 2y - 4x/y^3$$
$$\frac{\partial N_1}{\partial x} = 1 + 0 - 4/y^3$$
$$= 1 - 4/y^3$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

$\therefore$ Equation (2) is Exact Differential Equation

$u_1 + \int \phi(y)\, dy = C \longrightarrow$ (3) is the General Solution of eqn (2)

$$u_1 = \int M_1\, dx$$
$$= \int (y + 2/y^2)\, dx = y\int 1 dx + 2/y^2 \int 1 dx$$
$$= xy + 2x/y^2$$

$\phi_1(y)dy$ = No. of. terms of N_1 not contains (x)

$$\phi_1(y) = 2y$$
$$\phi_1(y)dy = 2\int y\, dy = \cancel{2}y^2/\cancel{2}$$

from (3)

$$= y^2$$

$$\boxed{xy + 2x/y^2 + y^2 = C}$$

(3) Solve $x^2 y\, dx - (x^3 + y^3)\, dy = 0$

Sol Given that $x^2 y\, dx - (x^3 + y^3)\, dy = 0 \longrightarrow$ (1)

$$x^2 \cdot y\, dx = (x^3 + y^3)\, dy$$
$$\frac{dy}{dx} = \frac{x^2 y}{x^3 + y^3} \longrightarrow (2)$$
$$f(x,y) = \frac{x^2 y}{x^3 + y^3}$$
$$f(kx + ky) = \frac{(kx)^2 \cdot (ky)}{(kx)^3 + (ky)^3}$$

$$= \frac{\cancel{k^3}(x^2 \cdot y)}{\cancel{k^3}(x^3+y^3)}$$

$$= k^0 \left[\frac{x^2 \cdot y}{x^3+y^3}\right]$$

$$= k^0 \left[\frac{x^2 \cdot y}{x^3+y^3}\right]$$

Clearly eqn 1 is H.D.E

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \longrightarrow 3$$

from eqn 2 & 3

$$v + x\,dv/dx = \frac{x^2 \cdot vx}{x^3 + (vx)^3}$$

$$= \frac{\cancel{x^3}\, v}{(1+v^3)\cancel{x^3}}$$

$$= \frac{v}{1+v^3}$$

$$v + x\frac{dv}{dx} = \frac{v}{1+v^3}$$

$$x\frac{dv}{dx} = \frac{v}{1+v^3} - v$$

$$= \frac{\cancel{v} - \cancel{v} - v^4}{1+v^3}$$

$$x\frac{dv}{dx} = \frac{-v^4}{1+v^4}$$

By Separating Variable Method

$$\left[\frac{(1+v^3)}{v^4}\right] dv = \frac{-dx}{x}$$

$$(v^{-4} + 1/v)dv = -1/x \, dx$$

Integrating on both sides

$$\int v^{-4} dv + \int 1/v \, dv = -\int 1/x \, dx$$

$$\frac{v^{-4+1}}{-4+1} + \log v = -\log x + c$$

$$-1/3 \, v^{-3} + \log v + \log x = c$$

$$-1/3 \cdot \frac{1}{(y/x)^3} + \log(y/x \cdot x) = c \qquad \because y = vx, \; v = y/x$$

$$\frac{-x^3}{3y^3} + \log y = c$$

$$\boxed{\log y - x^3/3y^3 = c}$$

(4) Solve $[1 + e^{x/y}] \, dx + e^{x/y} [1 - x/y] \, dy = 0$

Sol Given that,

$$(1 + e^{x/y}) dx + e^{x/y} [1 - x/y) \, dy = 0 \longrightarrow (1)$$

Equation (1) Compare with $Mdx + Ndy = 0$

$M = 1 + e^{x/y}$

$\frac{\partial M}{\partial y} = 0 + e^{x/y} [x(-1/y^2)]$

$= \frac{-xe^{x/y}}{y^2}$

$N = (1 - x/y)e^{x/y} = e^{x/y} - [e^{x/y} . x/y]$

$\frac{\partial N}{\partial x} = e^{x/y}(1/y) - [x/y . e^{x/y} + e^{x/y} . 1/y]$

$= e^{x/y} . 1/y - \frac{xe^{x/y}}{y^2} - e^{x/y} . 1/y$

$= \frac{-xe^{x/y}}{y^2}$

$\boxed{\therefore \frac{d}{dx} e^{ax} = ae^{ax}}$

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No. Name Date. Page No. 7

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Clearly Equation (1) is Exact Differential Equation

$\therefore u + \int \phi_y \, dy = C \longrightarrow$ (2) is a General Soln of eqn (1)

$u = \int_{(y)} M \, dx$

ϕ_y = Terms of 'N' not Containing 'x'

$$u = \int M dx = \int 1 + e^{x/y} dx$$

$$= \int 1 dx + \int e^{x/y} dx$$

$$= x + \frac{e^{x/y}}{(1/y)}$$

$$u = x + ye^{x/y}$$

$$\phi_y = 0$$

from (2)

$$x + ye^{x/y} + \int 0 \, dy = c$$

$$\boxed{\Rightarrow x + ye^{x/y} = \underline{\underline{c}}}$$