

# Certificate

Department of : Mathematics

Class: 1<sup>st</sup> BSc MPCS

Register No.: 221107102001



ertified that this is the bonafide record of practical work done in the laboratory by the candidate

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Examiners

# Index ...

Sr. No.	Name of the Experiment	Page No.	Date of Experiment	Date of Submission	Remarks
<u>DIFFERENTIAL EQUATIONS</u>					
1.	Differential Equations of first order and first degree.	1-8			
2.	Differential Equations of first order but not of first degree.	9 <sup>-15</sup> <del>10-16</del>			
3.	Higher order linear differential Equations - I	16-24			
4.	Higher order linear differential Equations - II	25-33			
5.	Higher order linear differential Equations - III.	34-44			

UNIT - 1 :- DIFFERENTIAL EQUATIONS OF FIRST ORDER AND FIRST DEGREE

Expt. No. 01.

Name

Date

Page No. 1.

1.  $(x^2y - 2xy^2) dx - (x^3 - 3x^2y) dy = 0.$

Sol: Given  $(x^2y - 2xy^2) dx - (x^3 - 3x^2y) dy = 0 \rightarrow (1)$

Eq (1) comparing with  $M dx + N dy = 0$

$M = x^2y - 2xy^2$

$N = -(x^3 - 3x^2y)$

$\frac{\partial M}{\partial y} = x^2 - 2 \cdot 2xy = x^2 - 4xy$

$\frac{\partial N}{\partial x} = -3x^2 + 3x^2 = 0$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

Eq (1) Non-Exact Differential Eqn & Homogeneous D.E.

Integration factor  $= \frac{1}{Mx + Ny}$

$Mx + Ny = (x^2y - 2xy^2)x + (-x^3 + 3x^2y)y$   
 $= x^3y - 2x^2y^2 - x^3y + 3x^2y^2$

I.F =  $\frac{1}{3x^2y^2 - 2x^2y^2}$

$= \frac{1}{x^2/y^2}$

Eq (1) multiply with I.F.

$\left[ \frac{x^2y - 2xy^2}{x^2y^2} dx + \frac{-x^3 + 3x^2y}{x^2y^2} dy \right] = 0.$

$= \left( \frac{x^2y}{x^2y^2} - \frac{2xy^2}{x^2y^2} \right) dx + \left( \frac{-x^3}{x^2y^2} + \frac{3x^2y}{x^2y^2} \right) dy = 0$

$\left( \frac{1}{y} - \frac{2}{x} \right) dx + \left( \frac{-x}{y^2} + \frac{3}{y} \right) dy = 0 \rightarrow (2)$

Again eq (2) compare with  $M_1 dx + N_1 dy = 0.$

$M_1 = \frac{1}{y} - \frac{2}{x}$

$N_1 = \frac{-x}{y^2} + \frac{3}{y}$

$\frac{\partial M_1}{\partial y} = -\frac{1}{y^2}$

$\frac{\partial N_1}{\partial x} = -\frac{1}{y^2}$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Eq (2) is E.D.E

$\therefore M + \int \phi(y) dy = c \rightarrow$  (3) is a general solution of eq (2)

$$M = \int M dx = \int \left( \frac{1}{y} - \frac{2}{x} \right) dx$$

$$= \frac{1}{y} \int 1 dx - 2 \int \frac{1}{x} dx$$

$$= \frac{x}{y} - 2 \log x$$

$\phi(y)$  = Term of  $N$  not containing  $x$

$$= 3/y.$$

from (3)

$$M + \int \phi(y) dy = c$$

$$x/y - 2 \log x + 3 \int \frac{1}{y} dy = c.$$

$$\Rightarrow \boxed{x/y - 2 \log x + 3 \log y = c}.$$

$$2) (y^4 + 2y) dx + (xy^3 + 2y^4 - 4x) dy = 0.$$

Sol: Given that  $(y^4 + 2y) dx + (xy^3 + 2y^4 - 4x) dy = 0.$

$$M = y^4 + 2y$$

$$N = xy^3 + 2y^4 - 4x$$

$$\frac{\partial M}{\partial y} = 4y^3 + 2$$

$$\frac{\partial N}{\partial x} = y^3 + 0 - 4 = y^3 - 4$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

eq (1) is NEDE, NHDE &  $T(M) < T(N)$

$$I.F = \int f(x) dy \quad \int e^{K dy}$$

$$f(y) = \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M}$$

$$= \frac{y^3 - 4 - 4y^3 - 2}{y^4 + 2y}$$

$$= \frac{-3y^3 - 6}{y^4 + 2y} = \frac{-3(y^3 + 2)}{y(y^3 + 2)} = -3/y.$$

$$e^{\int -3/y dy} = e^{-3 \int 1/y dy} = e^{-3 \log y}$$

$$= e^{-\log(y^3)}$$

$$= 1/y^3$$

eq ① multiply with I.F  $= 1/y^3$

$$1/y^3 (y^4 + 2y) dx + 1/y^3 (xy^3 + 2y^4 - 4xy) dy = 0$$

compare eq ② with  $M dx + N dy = 0$ .

$$M = y + 2/y^2$$

$$N = x + 2y - 4x/y^3$$

$$\frac{\partial M}{\partial y} = 1 + 2(-2y^{-3})$$

$$\frac{\partial N}{\partial x} = 1 + 0 - \frac{4}{y^3}$$

$$= 1 - \frac{4}{y^3}$$

$$1 - \frac{4}{y^3}$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\therefore$  eq ② is P.D.E.

$\mu + \int \phi y dy = c \rightarrow$  ③ is the general solution of ②

$$\mu = \int M dx$$

$$= \int (y + 2/y^2) dx = y \int 1 dx + 2/y^2 \int 1 dx$$

$$= xy + 2x/y^2$$

$\phi y =$  No. of terms of  $M$  not containing  $x$

$$\phi y = 2y$$

$$\int \phi y dy = 2 \int y dy$$

$$\text{eq ③} \Rightarrow xy + 2x/y^2 + y^2 = C.$$

$$3) x^2 y dx - (x^3 + y^3) dy = 0.$$

Sol: Given that  $(x^2 y) dx - (x^3 + y^3) dy = 0 \rightarrow$  ①

Eq ① compare with  $M dx + N dy = 0$ .

$$M = x^2 y$$

$$N = -x^3 - y^3$$

$$\frac{\partial M}{\partial y} = x^2$$

$$\frac{\partial N}{\partial x} = -3x^2$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

Eq ① is NEDE

from ①  $x^2y dx - (x^3+y^3)dy$

$$\frac{dy}{dx} = \frac{x^2y}{x^3+y^3}$$

$$\text{let } f(x,y) = \frac{x^2y}{x^3+y^3}$$

Put  $x = kx$ ,  $y = ky$ .

$$f(kx, ky) = \frac{(kx)^2(ky)}{(kx)^3+(ky)^3}$$

$$= k^0 \frac{x^2y}{x^3+y^3}$$

$$f(kx, ky) = k^0 f(x, y)$$

H.D.E of eq ① is NEDE & NHDE

Degree of  $k=0$ .

$$\text{I.F} = \frac{1}{Mx+Ny}$$

$$Mx+Ny = x^2y(x) + (-x^3-y^3)y$$

$$= x^3/y - x^3/y - y^4$$

$$= Mx+Ny = -y^4$$

$$\text{I.F} = \frac{1}{Mx+Ny} = \frac{-1}{y^4}$$

eq ① multiply with I.F =  $-\frac{1}{y^4}$

$$x^2y dx - [x^3+y^3] dy = 0$$

$$\left(-\frac{x^2y}{y^4}\right) dx + \left(\frac{x^3}{y^4} + \frac{y^3}{y^4}\right) dy = 0$$

$$-\frac{x^2}{y^3} dx + \left(\frac{x^3}{y^4} + \frac{1}{y}\right) dy = 0 \rightarrow \textcircled{2}$$

Again eq ② compare with  $M_1 dx + N_1 dy = 0$ .

$$M_1 = \frac{x^2}{y^3}$$

$$N_1 = \frac{x^3}{y^4} + \frac{1}{y}$$

$$\frac{\partial M_1}{\partial y} = -x^2$$

$$\frac{\partial N_1}{\partial x} = \frac{3x^2}{y^4}$$

$$\frac{\partial M_1}{\partial y} = \frac{3x^2}{y^4}$$

$$\frac{\partial N_1}{\partial x} = \frac{3x^2}{y^4}$$

$$\therefore \frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

Eq (2) is an E.D.E.

$u + \int \phi(y) dy = c$  is a general solution of eq (2)

$$M_1 = \int M_1 dx = \int \frac{-x^2}{y^3} dx$$

$$= \frac{-1}{y^3} \int x^2 dx$$

$$= \frac{-1}{y^3} \left( \frac{x^3}{3} \right)$$

$$= \frac{-x^3}{3y^3}$$

$$\phi(y) = \text{terms of } N_1 \text{ not contains } x.$$

$$= \frac{1}{y}$$

from (3)

$$-\frac{x^3}{3y^3} + \int \frac{1}{y} dy = c$$

$$\boxed{= \frac{-x^3}{3y^3} + \log y = c}$$

# Certificate

Department of : MATHEMATICS

Class: I<sup>st</sup> B.S.C [M.P.C] Register No.: 221107102002

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Examiners

# UNIT - 1

Expt. No.

Name

Date.

Page No. 1

## DIFFERENTIAL EQUATIONS OF FIRST ORDER AND FIRST DEGREE

1.  $[1 + e^{x/y}] dx + [1 - x/y] e^{x/y} dy = 0$

Sol. Given that

$$1 + e^{x/y} dx + [1 - x/y] e^{x/y} dy = 0 \quad \text{--- (1)}$$

Eqn compare with  $mdx + ndy = 0$

$$M = 1 + e^{x/y}$$

$$N = (1 - x/y) e^{x/y}$$

$$\begin{aligned} \frac{\partial M}{\partial y} &= 0 + e^{x/y} [x(1 - 1/y^2)] \\ &= \frac{-x e^{x/y}}{y^2} \end{aligned}$$

$$N = (1 - x/y) e^{x/y} = e^{x/y} - (x/y) e^{x/y}$$

$$\frac{\partial N}{\partial x} = e^{x/y} \left(\frac{1}{y}\right) - \left[\frac{x}{y} \cdot e^{x/y} \cdot \frac{1}{y} + e^{x/y} \cdot \frac{1}{y}\right]$$

$$\begin{aligned} &= \frac{e^{x/y}}{y} - \frac{x e^{x/y}}{y^2} - \frac{e^{x/y}}{y} \\ &= \frac{-x e^{x/y}}{y^2} \end{aligned}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \text{Given eqn 1 is B.P.E}$$

$\therefore \mu + \int \phi(y) dy = C \rightarrow C$  is a General solution of eqn

$$\mu = \int m dx$$

(y)

$\phi(y)$  = Terms of 'N' not containing 'x'

$$\mu = \int m dx = \int [1 + e^{x/y}] dx$$

$$= \int 1 dx + \int e^{x/y} dx$$

$$x + \frac{e^{x/y}}{(1/y)}$$

$$\mu = x + y e^{x/y}$$

$$\phi \cdot y = 0 \quad \text{from 2}$$

$$x + ye^{x/y} + \int \text{ody} = c$$

$$x + ye^{x/y} = c$$

Q  $x^2 \cdot y dx - (x^3 + y^3) dy = 0$

Sol Given that

$$x^2 \cdot y dx - (x^3 + y^3) dy = 0 \quad \text{--- (1)}$$

Given eqn compare with  $Mdx + Ndy = 0$

$$M = x^2 y$$

$$N = -x^3 - y^3$$

$$\frac{\partial M}{\partial y} = x^2$$

$$\frac{\partial N}{\partial x} = -3x^2$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

eqn 1 is NEDE from 1

$$x^2 y dy = (x^3 + y^3) dx$$

$$\frac{dy}{dx} = \frac{x^2 y}{x^3 + y^3}$$

$$\text{let } f(x, y) = \frac{x^2 y}{x^3 + y^3}$$

put  $x = kx$ , put  $y = ky$ .

$$f(kx, ky) = \frac{(kx)^2 (ky)}{(kx)^3 + (ky)^3}$$

$$k^0 \frac{x^2 y}{x^3 + y^3}$$

$$f(kx, ky) = k^0 f(x, y)$$

H.O.E eqn 1 is N.E.D.E & H.O.E degree of  $k=0$

$$I.F = \frac{1}{Mx + Ny}$$

$$\begin{aligned} Mx + Ny &= x^2y(x) + (-x^3 - y^3)y \\ &= x^3y - x^3y - y^4 \\ &= Mx + Ny = -y^4 \end{aligned}$$

$$I.F = \frac{1}{Mx + Ny} = -\frac{1}{y^4}$$

eqn 1 multiplying with  $I.F = -\frac{1}{y^4}$

$$x^2y dx - [x^3 + y^3] dy = 0$$

$$\left(-\frac{x^2y}{y^4}\right) dx + \left(\frac{x^3}{y^4} + \frac{y^3}{y^4}\right) dy = 0$$

$$-\frac{x^2}{y^3} dx + \left(\frac{x^3}{y^4} + \frac{1}{y}\right) dy = 0 \rightarrow 2$$

Again eqn 2 compare with

$$M_1 dx + N_1 dy = 0$$

$$M_1 = \frac{x^2}{y^3} \quad N_1 = \frac{x^3}{y^4} + \frac{1}{y}$$

$$\frac{\partial M_1}{\partial y} = -x^2$$

$$N_1 = \frac{3x^2}{y^4}$$

$$\frac{\partial M_1}{\partial y} = \frac{3x^2}{y^4}$$

$$\frac{\partial N_1}{\partial x} = \frac{3x^2}{y^4}$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

Eqn 2 is an E.D.E

$M_1 + S\phi_1(y)dy = c$  is a General sol of eqn =

$$u = \int m_1 dx = \int \frac{-x^2}{y^3} dx$$

$$\frac{-1}{y^3} \int x^2 dx$$

$$\frac{-1}{y^3} \left[ \frac{x^2+1}{2+1} \right]$$

$$= \frac{-x^3}{3y^3}$$

$q_1(y)$  = Term of  $N_1$  not containing  $x = 1/y$   
from 3

$$\frac{-x^3}{3y^3} + \int \frac{1}{y} dy = c$$

$$\frac{-x^3}{3y^3} + \log y = c$$

$$3. (x^2y - 2xy^2) dx - (x^3 - 3x^2y) dy = 0$$

$$\text{sol } (x^2y - 2xy^2) dx - (x^3 - 3x^2y) dy = 0 \rightarrow 1$$

Eqn 1 compare with

$$Mdx + Ndy = 0$$

$$M = x^2y - 2xy^2$$

$$N = -(x^3 - 3x^2y)$$

$$\frac{\partial M}{\partial y} = x^2 - 2 \cdot x \cdot 2y$$

$$\frac{\partial N}{\partial x} = -3x^2 + 3x^2$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

Eqn 1 Non-Exact differential equation  
& Homogeneous differential equation.  
Integration factor.

$$Mx + Ny$$

$$Mx + Ny = (x^2y - 2xy^2)x + (-x^3 + 3x^2y)y.$$

$$x^3/y - 2x^2y^2 - x^3/y + 3x^2y^2$$

$$I.F = \frac{1}{3x^2y^2 - 2x^2y^2} = \frac{1}{x^2y^2(3-2)}$$

$$\frac{1}{x^2y^2}$$

eqn 1 multiply with I.F

$$\left[ \frac{x^2y - 2xy^2}{x^2y^2} \right] dx + \left[ \frac{-x^3 + 3x^2y}{x^2y^2} \right] dy = 0$$

$$\left[ \frac{x^2y}{x^2y^2} - \frac{2xy^2}{x^2y^2} \right] dx + \left[ \frac{-x^3}{x^2y^2} + \frac{3x^2y}{x^2y^2} \right] dy = 0$$

$$\left[ \frac{1}{y} - \frac{2}{x} \right] dx + \left[ \frac{x}{y^2} + \frac{3}{y} \right] dy = 0 \rightarrow 2$$

Again eqn 2 compare with

$$M_1 dx + N_1 dy = 0$$

$$M_1 = \frac{1}{y} - \frac{2}{x}$$

$$N_1 = \frac{x}{y^2} + \frac{3}{y}$$

$$\frac{\partial M_1}{\partial y} = -\frac{1}{y^2}$$

$$\frac{\partial N_1}{\partial x} = \frac{1}{y^2}$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

eqn 2 is E.D.E

$$M_1 + \int \phi(y) dy = c \rightarrow 3 \text{ is a}$$

General solution of eqn 2

Date.

Expt. No.

Name

$$= y \frac{3y - 4y^3 - 2}{y^4 + 2y}$$

$$= \frac{-3y^3 - 6}{y^4 + 2} = \frac{-3(y^3 + 2)}{y(y^3 + 2)} = -3/y$$

$$e^{\int -3/y dy} = e^{-3 \int 1/y dy} = e^{-3 \log y}$$

$$= e^{\log (1/y^3)}$$

$$= 1/y^3$$

eqn 1 multiply with T.F =  $1/y^3$

$$1/y^3 (y^4 + 2y) dx + 1/y^3 (xy^3 + 2y^4 - 4x) dy = 0$$

$$(y + \frac{2}{y^2}) dx + (x + 2y - 4x/y^3) dy = 0.$$

Compare eqn 2 with  $M dx + N dy = 0$

$$M_1 = y + \frac{2}{y^2} \quad N_1 = x + 2y - 4x/y^3$$

$$\frac{\partial M_1}{\partial y} = 1 + 2(-2y^{-3}) \quad \frac{\partial N_1}{\partial x} = 1 + 0 - 4/y^3$$

$$= 1 - \frac{4}{y^3}$$

$$1 - \frac{4}{y^3}$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

$\therefore$  eqn 2 is E.O.E

$u + \int \phi y dy = c \rightarrow 3$  is a General Solution  
of eqn 2

$$u = \int m dx$$

$$\int (y + \frac{2}{y^2}) dx = y \int 1 dx + \frac{2}{y^2} \int 1 dx$$

$$xy + \frac{2x}{y^2}$$

$\phi y = \text{NO}$  of terms of  $N_1$  not  
contains  $x$

~~$$\phi y = 2y$$~~

$$\phi y dy = 2 \int y dy$$

~~$$\text{eqn 3} \Rightarrow xy + \frac{2x}{y^2} + y^2 = c$$~~

# Certificate

Department of : MATHEMATICS

Class: I<sup>st</sup> BSC [MPC] Register No.: 221107102003

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Examiners

# Index ...

## DIFFERENTIAL EQUATIONS

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3.	Higher Order Linear Differential Equations - I	17-25			
4.	Higher Order linear Differential Equations - II	26-34			
5.	Higher Order linear Differential Equations - III	35-44			

# UNIT-1 :- DIFFERENTIAL EQUATIONS OF FIRST ORDER

B1 FIRST DEGREE

Expt. No. 01

Name

Date

Page No. 1

1.  $(x^2y - 2xy^2) dx - (x^3 - 3x^2y) dy = 0$

②  $(x^2y - 2xy^2) dx - (x^3 - 3x^2y) dy = 0 \rightarrow 0$

Egn 1 Comparing with

$M dx + N dy = 0$

$M = x^2y - 2xy^2$        $N = -(x^3 - 3x^2y)$

$\frac{\partial M}{\partial y} = x^2 - 2xy$        $= -x^3 + 3x^2y$

$\frac{\partial N}{\partial x} = -3x^2 + 3x^2$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Egn 1 Non-exact Differential Egn & Homogeneous D.E

Note: Variable separable =  $\frac{M}{N} = \frac{dx}{dy}$

$M dx + N dy = (x^2y - 2xy^2) dx + (-x^3 + 3x^2y) dy$

$= x^2y dx - 2xy^2 dx - x^3 dy + 3x^2y dy$

$I f = \frac{1}{3x^2y - 2xy^2} = \frac{1}{x^2y(3 - 2y)}$

$= \frac{1}{x^2y}$

Egn 1 multiply with I.f

$\left[ \frac{x^2y - 2xy^2}{x^2y} \right] dx + \left[ \frac{-x^3 + 3x^2y}{x^2y} \right] dy = 0$

$\left[ \frac{x^2y}{x^2y} - \frac{2xy^2}{x^2y} \right] dx + \left[ \frac{-x^3}{x^2y} + \frac{3x^2y}{x^2y} \right] dy = 0$

$\left[ \frac{1}{y} - \frac{2}{x} \right] dx + \left[ \frac{-x}{y} + 3 \right] dy = 0 \rightarrow 0$

Again eqn 2 compare with  $M dx + N dy = 0$

$$M_1 = \frac{1}{y} = \frac{1}{y^2} \quad N_1 = -\frac{x}{y} + \frac{3}{y}$$

$$\frac{\partial M_1}{\partial y} = -\frac{1}{y^3} \quad \frac{\partial N_1}{\partial x} = -\frac{1}{y^2}$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

Eqn 2 is E.D.F

$$\therefore M_1 + \int \phi_1(y) dy = C \rightarrow 3.15 a$$

Ans of eqn 2

$$u_1 = \int m dx = \int \left( \frac{1}{y} - \frac{x}{y} \right) dx$$

$$= \frac{1}{y} \int dx - \frac{x}{y} dx$$

$$= \frac{x}{y} - \frac{x^2}{2y}$$

$\phi_1(y) =$  Terms of  $N$  not containing  $x = \frac{3}{y}$

from (3)

$$u_1 + \int \phi_1(y) dy = C$$

$$\frac{x}{y} - \frac{x^2}{2y} + 3 \int \frac{1}{y} dy = C$$

$$\left[ \frac{x}{y} - \frac{x^2}{2y} + 3 \log y - C \right]$$

(3)

$$(y^4 + 2y) dx + (xy^3 + 2y^4 - 4yx) dy = 0$$

A) Given that  $(y^4 + 2y) dx + (xy^3 + 2y^4 - 4yx) dy = 0$

Eqn 1 compare with  $M dx + N dy = 0$

$$M = y^4 + 2y \quad N = xy^3 + 2y^4 - 4yx$$

$$\frac{\partial M}{\partial y} = 4y^3 + 2 \quad \frac{\partial N}{\partial x} = y^3 + 0 - 4 = y^3 - 4$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

eqn 1 is NEDE, NHDE & TCM) < T(N)

I.F. =  $\int f(x) dy$   $\int t dy$ .

$$f(y) = \left[ \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right]_{CM}$$

$$= \frac{y^3 - 4 - 4y^3 - 2}{y^4 + 2y}$$

$$= \frac{-3y^3 - 6}{y^4 + 2} = \frac{-3(y^3 + 2)}{y^4 + 2} = \frac{-3}{y}$$

$$e^{\int -3/y dy} = e^{-3 \int 1/y dy} = e^{-3 \log y} \\ = e^{\log (1/y^3)} \\ = 1/y^3$$

eqn 1 multiply with I.F. =  $1/y^3 (xy^3 + 2y^4 - 4x) dy = 0$

$$1/y^3 (y^4 + 2y) dx + 1/y^3 (xy^3 + 2y^4 - 4x) dy = 0 \\ (y + 2/y^2) dx + (x + 2y - 4x/y^3) dy = 0$$

Compare eqn 2 with  $M dx + N dy = 0$

$$M_1 = y + 2/y^2 \quad N_1 = x + 2y - 4x/y^3$$

$$\frac{\partial M_1}{\partial y} = 1 + 2(-2y^{-3}) \quad \frac{\partial N_1}{\partial x} = 1 + 0 - 4/y^3 \\ = 1 - 2/y^3 \quad \frac{\partial N_1}{\partial x} = 1 - 2/y^3$$

$\therefore$  eqn 2 is I.F.D.F

$$M_1 + \int \frac{\partial M_1}{\partial y} dy = C \rightarrow 3. \text{ I.F.S of eqn 2}$$

$$M_1 = \int M_1 dx$$

$$= \int (y + 2/y^2) dx - y \int dx + 2/y^2 \int dx \\ = xy + 2x/y^2$$

$\phi y$  - A no. of terms of  $N_1$  Not Contains  $x$   
 $\phi y = 2y$

$$\int p y \, dy = q \int y \, dy$$

$$\text{eqn 2} \Rightarrow xy + \frac{2x}{y} + y^2 = C$$

3)  $x^2 - y \, dx - (x^2 + y^2) \, dy = 0$

A) Let  $x^2 - y \, dx - (x^2 + y^2) \, dy = 0 \quad \text{--- (1)}$

Given eqn (1) compare with

$$M \, dx + N \, dy = 0$$

$$M = x^2 - y \quad N = -x^2 - y^2$$

$$\frac{\partial M}{\partial y} = -1 \quad \frac{\partial N}{\partial x} = -2x$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

eqn (1) is N.F.D.E

$$\text{from (1) } x^2 y \, dx = (x^2 + y^2) \, dy$$

$$\frac{dy}{dx} = \frac{x^2 + y^2}{x^2}$$

Let  $f(x, y) = \frac{x^2 y^3}{x^2 + y^3}$

Put  $x = kx, y = ky$

$$f(kx, ky) = (kx)^2 (ky)^3 / (kx)^2 + (ky)^3$$

$$= k^0 \frac{x^2 y^3}{x^2 + y^3}$$

$$f(kx, ky) = k^0 f(x, y)$$

H.D.F eqn (1) is N.F.D.E & H.D.F

degree of  $k = 0$

$$I.F = \frac{1}{x^2 y^3}$$

$$Mx + Ny = x^2y(x) + (x^2 - y^3)y$$

$$= x^3y - x^2y - y^4$$

$$Mx + Ny = -y^4$$

$$T.F. = \frac{1}{Mx + Ny} = -\frac{1}{y^4}$$

Eqn 1 multiplying with T.F. = -1/y^4

$$x^2y dx - [x^2 + y^3] dy = 0$$

$$\left[ \frac{-x^3y}{y^4} \right] dx + \left[ \frac{x^2}{y^4} + \frac{y^3}{y^4} \right] dy = 0$$

$$\frac{-x^3}{y^3} dx + \left[ \frac{x^2}{y^4} + \frac{1}{y} \right] dy = 0 \rightarrow (2)$$

Again eqn 2 compare with

$$M_1 dx + N_1 dy = 0$$

$$M_1 = \frac{-x^3}{y^3} \quad N_1 = \frac{x^2}{y^4} + \frac{1}{y}$$

$$\frac{\partial M_1}{\partial y} = \frac{-x^3}{y^4} \quad N_1 = \frac{-2x^2}{y^5} + \frac{-1}{y^2}$$

$$\frac{\partial M_1}{\partial y} = \frac{-2x^2}{y^5} + \frac{-1}{y^2} \quad \frac{\partial N_1}{\partial x} = \frac{2x}{y^5}$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

Eqn 2 is an E.D.F

M, + f(x, y) dy = c is a general sol of eqn 2

$$M_1 dx = \int -\frac{x^3}{y^3} dx$$

$$= -\frac{1}{y^3} \int x^3 dx$$

$$= -\frac{1}{y^3} \left( \frac{x^4}{4} + C \right)$$

$$= \frac{-x^3}{3y^3}$$

Q. (y) = Term of N, Mod

$\frac{1}{y}$

from (3)

$$\frac{-x^3}{3y^3} + \int \frac{1}{y} dy = c$$

$$\frac{-x^3}{3y^3} + \log y = c$$

(4)  $x \frac{dy}{dx} + y = y^x \log x$

Let  $x \frac{dy}{dx} + y = y^x \log x$

$$\frac{dy}{dx} + \frac{y}{x} = \frac{y^x \log x}{x} \rightarrow 0$$

eqn 1 is BNE in y

$$P(x) = \frac{1}{x} \log(x) = \frac{\log x}{x}, n=2$$

eqn 1 divided with  $y^n$

$$\frac{1}{y} \frac{dy}{dx} + \frac{1}{x} = \frac{1}{y} \log x = \frac{\log x}{x}$$

$$y^{-2} \frac{dy}{dx} + \frac{1}{x} y^{-1} = \frac{\log x}{x} \rightarrow 0$$

$$y^{-1} = t$$

$$-1 \cdot y^{-1-1} \frac{dy}{dx} = \frac{dt}{dx}$$

$$y^{-2} \frac{dy}{dx} = -\frac{dt}{dx} \rightarrow 0$$

from (3) & (4)

$$-\frac{dt}{dx} + \frac{1}{x} \cdot t = \frac{\log x}{x}$$

$$\frac{dt}{dx} + (-1/x) \cdot 1 = -\frac{1 \log x}{x} = 0$$

Eqn 4 is L.D.E in 't'

∴ t [Int] = ∫ f(x) dx + C is a G.S of eqn 4

$$I f = \int e^{\int p(x) dx}$$

$$= \int e^{\int 1/x dx}$$

$$= e^{-\log x}$$

$$= e^{\log x^{-1}}$$

$$= x^{-1} = 1/x$$

$$I f = 1/x$$

from u

$$1/x = \int -\frac{1 \log x}{x} \cdot 1/x dx + C = -\int x^{-2} \log x dx + C$$

$$= -\int x^{-2} \log x dx + C = -\log x \int x^{-2} dx - \int \left( \frac{1}{x} \cdot \int x^{-2} dx \right) dx + C$$

$$= -\int \log x \left[ \frac{x^{-2+1}}{-2+1} \right] - \int \frac{1}{x} \left[ \frac{x^{-2+1}}{-2+1} + 1 \right] dx + C$$

$$= \left[ \frac{\log x}{x} - \int x^{-2} dx \right] + C$$

$$+ 1/x = \frac{\log x}{x} + 1/x + C$$

$$y^{-1} - 1/x = \frac{\log x + 1}{x} + C$$

$$1/x y = \frac{\log x + 1}{x} + C$$

# Certificate

Department of: MATHEMATICS

Class: I<sup>st</sup> BSC [PG] Register No.: 221107102004



Certified that this is the bonafide record of practical

work done in the laboratory by the candidate

*Kuvivella Vyshnavi*

during the year 2022 - 2023

No. of Practicals conducted 05

No. of Practicals attended 05

*[Signature]*  
Lecturer

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Submitted for the practical examination held on

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2. ....

Examiners

# Index ...

Sr. No.	Name of the Experiment	Page No.	Date of Experiment	Date of Submission	Remarks
	<b>* DIFFERENTIAL EQUATIONS *</b>				
1.	Differential Equation of first order and first degree	1-7			
2.	Differential Equations of first order but not of the first degree	10-17			
3.	Higher Order Linear Differential equations - I	18-24			
4.	Higher Order Linear Differential equations - II	25-33			
5.	Higher Order Linear Differential Equations - III	34-45			

# DIFFERENTIAL EQUATIONS

## UNIT-1 - Differential equations of 1<sup>st</sup> order & 1<sup>st</sup> degree

### \* SOLVED PROBLEMS \*

1.  $(x^2y - 2xy^2) dx - (x^3 - 3x^2y) dy = 0$

Sol: Given that  $(x^2y - 2xy^2) dx - (x^3 - 3x^2y) dy = 0$   
eqn compare with  $Mdx + Ndy = 0$

$$M = x^2y - 2xy^2 \quad N = -(x^3 - 3x^2y)$$

$$\frac{\partial M}{\partial y} = x^2 - 2xy^2 \quad \frac{\partial N}{\partial x} = -3x^2 + 3x^2y$$

$$= x^2 - 4xy \quad \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

eqn is Non-Exact Differential equation & Homogeneous D.E.

Integrating factors =  $\frac{Mx + Ny}{y^2}$

$$Mx + Ny = (x^2y - 2xy^2)x + (-x^3 + 3x^2y)y$$

$$= x^3y - 2x^2y^2 - x^3y + 3x^2y^2$$

$$I.F = \frac{1}{3x^2y^2 - 2x^2y^2} = \frac{1}{x^2y^2(3-2)}$$

$$= \frac{1}{x^2y^2}$$

eqn multiply with I.F.

$$\left( \frac{x^2y - 2xy^2}{x^2y^2} \right) dx + \left( \frac{-x^3 + 3x^2y}{x^2y^2} \right) dy = 0$$

$$\left( \frac{x^2y}{x^2y^2} - \frac{2xy^2}{x^2y^2} \right) dx + \left( \frac{-x^3}{x^2y^2} + \frac{3x^2y}{x^2y^2} \right) dy = 0$$

$$\left(\frac{1}{y} - \frac{2}{x}\right) dx + \left(-\frac{x}{y^2} + \frac{3}{y}\right) dy = 0 \rightarrow \textcircled{2}$$

Again eqn<sup>②</sup> compare with  $M_1 dx + N_1 dy = 0$

$$M_1 = \frac{1}{y} - \frac{2}{x}$$

$$N_1 = -\frac{x}{y^2} + \frac{3}{y}$$

$$\frac{\partial M_1}{\partial y} = -1/y^2$$

$$\frac{\partial N_1}{\partial x} = \frac{1}{y^2}$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

eqn<sup>②</sup> is Exact Differential Equation

$\therefore \mu_1 + \int \phi_1 dy = C \rightarrow \textcircled{3}$  is a General Solution of eqn<sup>②</sup>

$$\begin{aligned} \mu_1 &= \int M_1 dx = \int \left(\frac{1}{y} - \frac{2}{x}\right) dx \\ &= \frac{1}{y} \int dx - 2 \int \frac{1}{x} dx \end{aligned}$$

$$= \frac{x}{y} - 2 \log x$$

$\phi_1(y) =$  Terms of 'N' not containing 'x'

$$= \frac{3}{y}$$

from<sup>③</sup>

$$\mu_1 + \int \phi_1 dy = C$$

$$\frac{x}{y} - 2 \log x + 3 \int \frac{1}{y} dy = C$$

$$\boxed{\frac{x}{y} - 2 \log x + 3 \log y = C}$$



$$M_1 = y + 2/y^2$$

$$\frac{\partial M_1}{\partial y} = 1 + 2(-2y^{-3})$$

$$= 1 - 4/y^3$$

$$N_1 = x + 2y - 4x/y^3$$

$$\frac{\partial N_1}{\partial x} = 1 + 0 - 4/y^3$$

$$= 1 - 4/y^3$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

$\therefore$  eqm (2) is Exact Differential Equation.

$M_1 + \int \phi_2 y dy = c \rightarrow (3)$  is the General soln of eqm (2)

$$M_1 = \int M_1 dx$$

$$= \int (y + \frac{2}{y^2}) dx = y \int dx + 2 \int \frac{1}{y^2} dx$$

$$= xy + 2x/y^2$$

$\phi_1(x,y) =$  No. of terms of  $N_1$  not contains 'x'

$$\phi_1(y) = 2y$$

$$\phi_1(y) dy = 2 \int y dy = \frac{2y^2}{2}$$

from (3)

$$= y^2$$

$$xy + 2x/y^2 + y^2 = c$$

3. Solve  $x^2 y dx - (x^3 + y^3) dy = 0$

soln: Given that  $x^2 y dx - (x^3 + y^3) dy = 0 \rightarrow (1)$

$$x^2 \cdot y dx = (x^3 + y^3) dy$$

$$\frac{dy}{dx} = \frac{x^2 y}{x^3 + y^3} \rightarrow (2)$$

$$f(x,y) = x^2 y$$

$$x^3 + y^3$$

$$f(kx + ky) = \frac{(kx)^2 \cdot (ky)}{(kx)^3 + (ky)^3}$$

$$(kx)^3 + (ky)^3$$

$$\begin{aligned}
 & \frac{y^3(x^2-y)}{y^3(x^3+y^3)} \\
 &= \frac{y^3(x^2-y)}{y^3(x^3+y^3)} \\
 &= k^0 \left[ \frac{x^2-y}{x^3+y^3} \right] \\
 &= k^0 \left[ \frac{x^2-y}{x^3+y^3} \right]
 \end{aligned}$$

clearly eqn (1) is H.D.E.

Put  $y = v^4$

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \rightarrow (2)$$

from eqn (2) & (3)

$$v + x \frac{dv}{dx} = \frac{x^2 \cdot v^4}{x^3 + (v^4)^3}$$

$$= \frac{x^2 v}{(1+v^3)x^3}$$

$$= \frac{v}{1+v^3}$$

$$v + x \frac{dv}{dx} = \frac{v}{1+v^3}$$

$$x \frac{dv}{dx} = \frac{v}{1+v^3} - v$$

$$= \frac{v - v - v^4}{1+v^3}$$

$$x \frac{dv}{dx} = \frac{-v^4}{1+v^3}$$

By Separating Variable Method

$$\left[ \frac{(1+v^3)}{v^4} \right] dv = \frac{-dx}{x}$$

$$(v^{-4} + \frac{1}{v}) dv = -\frac{1}{x} dx$$

Integrating on both sides

$$\int v^{-4} dv + \int \frac{1}{v} dv = -\int \frac{1}{x} dx$$

$$\frac{v^{-4+1}}{-4+1} + \log v = -\log x + c$$

$$-\frac{1}{3} v^{-3} + \log v + \log x = c$$

$$-\frac{1}{3} \cdot \frac{1}{(\frac{y}{x})^3} + \log \left( \frac{y}{x} \cdot x \right) = c$$

$$\because y = vx$$

$$v = y/x$$

$$-\frac{x^3}{3y^3} + \log y = c$$

$$\log y - \frac{x^3}{3y^3} = C_1$$

4. Solve  $[1 + e^{x/y}] dx + e^{x/y} [1 - x/y] dy = 0$

sol<sup>n</sup>: Given that,  $(1 + e^{x/y}) dx + e^{x/y} (1 - x/y) dy = 0 \rightarrow (1)$

eqn (1) compare with  $M dx + N dy = 0$

$$M = 1 + e^{x/y}$$

$$N = (1 - x/y) e^{x/y} = e^{x/y} - e^{x/y} \cdot \frac{x}{y}$$

$$\frac{\partial M}{\partial y} = 0 + e^{x/y} \left[ x(-\frac{1}{y^2}) \right] = \frac{\partial N}{\partial x} = e^{x/y} \left( \frac{1}{y} \right) - \left[ \frac{x}{y} \cdot e^{x/y} + e^{x/y} \cdot \frac{x}{y} \right]$$

$$= \frac{-x e^{x/y}}{y^2} = \frac{x e^{x/y}}{y} - x e^{x/y} \cdot \frac{1}{y} = \frac{-x e^{x/y}}{y^2}$$

$$= \frac{-x e^{x/y}}{y^2} = \frac{y^2}{y^2} \cdot \frac{-x e^{x/y}}{1} = \frac{d}{dz} e^{ax} = a e^{ax}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

clearly eqn (1) is Exact Differential Eqn.

$\therefore M + \int \phi_y dy = C - x e^{x/y}$  is a General sol<sup>n</sup> of eqn (1)

$$u = \int \frac{1}{y} dx$$

Expt. No.

Name

Date.

Page No. 7

$\phi_y =$  Terms of 'N' not containing 'x'

$$u = \int M dx = \int (1 + e^{x/y}) dx$$

$$\int e^{ax} = e^{ax}/a$$

$$= \int 1 dx + \int e^{x/y} dx.$$

$$= x + \frac{e^{x/y}}{(1/y)}$$

$$u = x + ye^{x/y}$$

$$\phi_y' = 0$$

from (2)


$$x + ye^{x/y} + \int 0 dy = c.$$

$$\Rightarrow x + ye^{x/y} = c //$$

# Certificate

Department of: MATHEMATICS

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ertified that this is the bonafide record of practical

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Examiners



# DIFFERENTIAL EQUATIONS

Expt. No.

Name

Date.

Page No. 1

## UNIT-1 Differential Equations of 1<sup>st</sup> order & 1<sup>st</sup> degree

### \* SOLVED PROBLEMS \*

$$\textcircled{1} (x^2y - 2xy^2) dx - (x^3 - 3x^2y) dy = 0$$

Sol: Given that

$$(x^2y - 2xy^2) dx - (x^3 - 3x^2y) dy = 0 \rightarrow \textcircled{1}$$

Equation ① Compare with  $Mdx + Ndy = 0$

$$M = x^2y - 2xy^2$$

$$N = -(x^3 - 3x^2y)$$

$$= -x^3 + 3x^2y$$

$$\frac{\partial M}{\partial y} = x^2 - 2xy^2$$

$$= x^2 - 4xy$$

$$\frac{\partial N}{\partial x} = -3x^2 + 3x^2y$$

$$= -3x^2 + 6xy$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

Equation ① Non-Exact Differential Equation & Homogeneous Differential Equation.

$$\text{Integrating factor} = \frac{1}{Mx + Ny}$$

$$Mx + Ny = (x^2y - 2xy^2)x + (-x^3 + 3x^2y)y$$
$$= (x^3y - 2x^2y^2 - x^3y + 3x^2y^2)$$

$$IF = \frac{1}{3x^2y^2 - 2x^2y^2} = \frac{1}{x^2y^2(3-2)}$$

$$= \frac{1}{x^2y^2}$$

Equation ① multiply with I.F

$$\left( \frac{x^2y - 2xy^2}{x^2y^2} \right) dx + \left( \frac{-x^3 + 3x^2y}{x^2y^2} \right) dy = 0$$

$$\left( \frac{x^2y}{x^2y^2} - \frac{2xy^2}{x^2y^2} \right) dx + \left( \frac{-x^3}{x^2y^2} + \frac{3x^2y}{x^2y^2} \right) dy = 0$$

$$\left(\frac{1}{y} - \frac{2}{x}\right) dx + \left(\frac{-x}{y^2} + \frac{3}{y}\right) dy = 0 \rightarrow (2)$$

Again eqn (2) Compare with  $M_1 dx + N_1 dy = 0$

$$M_1 = \frac{1}{y} - \frac{2}{x}$$

$$N_1 = \frac{-x}{y^2} + \frac{3}{y}$$

$$\frac{\partial M_1}{\partial y} = \frac{-1}{y^2}$$

$$\frac{\partial N_1}{\partial x} = \frac{-1}{y^2}$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

Equation (2) is Exact Differential Equation

$\therefore M_1 + \int \phi_1(x,y) dy = C \rightarrow (3)$  is a General Solution of eqn (2)

$$\begin{aligned} M_1 &= \int M_1 dx = \int \left(\frac{1}{y} - \frac{2}{x}\right) dx \\ &= \frac{1}{y} \int 1 dx - 2 \int \frac{1}{x} dx \\ &= \frac{x}{y} - 2 \log x \end{aligned}$$

$$\begin{aligned} \phi_1(x,y) &= \text{Terms of 'N}_1 \text{ not containing 'x'} \\ &= \frac{3}{y} \end{aligned}$$

from (3)

$$M_1 + \int \phi_1(x,y) dy = C$$

$$\frac{x}{y} - 2 \log x + 3 \int \frac{1}{y} dy = C$$

$$\boxed{\frac{x}{y} - 2 \log x + 3 \log y = C}$$

$$(2) (y^4 + 2y) dx + (xy^3 + 2y^4 - 4x) dy = 0$$

Sol: Given that  $(y^4 + 2y) dx + (xy^3 + 2y^4 - 4x) dy = 0 \rightarrow (1)$

Equation (1) Compare with  $M dx + N dy = 0$

$$M = y^4 + 2y$$

$$N = xy^3 + 2y^4 - 4x$$

$$\frac{\partial M}{\partial y} = 4y^3 + 2$$

$$\frac{\partial N}{\partial x} = y^3 + 0 - 4$$

$$= y^3 - 4$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

Equation (1) NEDE, NHDE and  $(CM) < (CN)$

$$I.F = e^{\int f(x) dx} \quad (or) \quad e^{\int S(y) dy}$$

$$f(y) = \frac{\left[ \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right]}{M}$$

$$= \frac{y^3 - 4 - 4y^3 - 2}{y^4 + 2y}$$

$$= \frac{-3y^3 - 6}{y^4 + 6} = \frac{-3(y^3 + 2)}{y(y^3 + 2)} = \frac{-3}{y}$$

$$e^{\int -3/y dy} = e^{-3 \int 1/y dy} = e^{-3 \log y}$$

$$= e^{\log (1/y^3)}$$

$$= 1/y^3$$

Equation (1) multiply with I.F =  $1/y^3$

$$\frac{1}{y^3} (y^4 + 2y) dx + \frac{1}{y^3} (xy^3 + 2y^4 - 4x) dy = 0$$

$$(y + 2/y^2) dx + (x + 2y - 4x/y^3) dy = 0 \rightarrow (2)$$

Equation (2) Compare with  $M dx + N dy = 0$

$$M_1 = y + 2/y^2$$

$$\frac{\partial M_1}{\partial y} = 1 + 2(-2y^{-3})$$

$$= 1 - 4/y^3$$

$$N_1 = x + 2y - 4x/y^3$$

$$\frac{\partial N_1}{\partial x} = 1 + 0 - 4/y^3$$

$$= 1 - 4/y^3$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

$\therefore$  Equation (2) is Exact Differential Equation

$M_1 + \int \phi_1(y) dy = C \rightarrow$  (3) is the General Solution of eqn (2)

$$M_1 = \int M_1 dx$$

$$= \int (y + 2/y^2) dx = y \int 1 dx + 2/y^2 \int 1 dx$$

$$= xy + 2x/y^2$$

$\phi_1(y) dy =$  No. of terms of  $N_1$  not contains 'x'

$$\phi_1(y) = 2y$$

$$\phi_1(y) dy = 2y dy = 2y^2/2$$

from (3)

$$= y^2$$

$$xy + 2x/y^2 + y^2 = C_1$$

(3) Solve  $x^2 y dx - (x^3 + y^3) dy = 0$

Sol Given that  $x^2 y dx - (x^3 + y^3) dy = 0 \rightarrow$  (1)

$$x^2 y dx = (x^3 + y^3) dy$$

$$\frac{dy}{dx} = \frac{x^2 y}{x^3 + y^3} \rightarrow$$
 (2)

$$f(x, y) = \frac{x^2 y}{x^3 + y^3}$$

$$f(kx + ky) = \frac{(kx)^2 \cdot (ky)}{(kx)^3 + (ky)^3}$$

$$= \frac{k^3 (x^2 \cdot y)}{k^3 (x^3 + y^3)}$$

$$= k^0 \left[ \frac{x^2 \cdot y}{x^3 + y^3} \right]$$

$$= k^0 \left[ \frac{x^2 \cdot y}{x^3 + y^3} \right]$$

Clearly eqn (1) is H.D.E

Put  $y = vx$

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \rightarrow (3)$$

from eqn (2) & (3)

$$v + x \frac{dv}{dx} = \frac{x^2 \cdot vx}{x^3 + (vx)^3}$$

$$= \frac{x^3 v}{(1+v^3)x^3}$$

$$= \frac{v}{1+v^3}$$

$$v + x \frac{dv}{dx} = \frac{v}{1+v^3}$$

$$x \frac{dv}{dx} = \frac{v}{1+v^3} - v$$

$$= \frac{v - v - v^4}{1+v^3}$$

$$x \frac{dv}{dx} = \frac{-v^4}{1+v^3}$$

By Separating Variable Method

Teacher's Signature

$$\left[ \frac{(1+v^3)}{v^4} \right] dv = \frac{-dx}{x}$$

$$(v^{-4} + 1/v) dv = -1/x dx$$

Integrating on both sides

$$\int v^{-4} dv + \int 1/v dv = -\int 1/x dx$$

$$\frac{v^{-4+1}}{-4+1} + \log v = -\log x + c$$

$$-1/3 v^{-3} + \log v + \log x = c$$

$$-1/3 \cdot \frac{1}{(y/x)^3} + \log(y/x \cdot x) = c$$

$$\because y = vx$$

$$v = y/x$$

$$-\frac{x^3}{3y^3} + \log y = c$$

$$\log y - \frac{x^3}{3y^3} = \underline{\underline{c}}$$

4) Solve  $[1 + e^{x/y}] dx + e^{x/y} [1 - x/y] dy = 0$

Sol Given that,

$$(1 + e^{x/y}) dx + e^{x/y} [1 - x/y] dy = 0 \rightarrow \text{①}$$

Equation ① Compare with  $Mdx + Ndy = 0$

$$M = 1 + e^{x/y}$$

$$N = (1 - x/y)e^{x/y} = e^{x/y} - [e^{x/y} \cdot x/y]$$

$$\frac{\partial M}{\partial y} = 0 + e^{x/y} [x(-1/y^2)]$$

$$\frac{\partial N}{\partial x} = e^{x/y} (1/y) - [x/y \cdot e^{x/y} + e^{x/y} \cdot 1/y]$$

$$\frac{\partial M}{\partial y} = \frac{-xe^{x/y}}{y^2}$$

$$= \frac{e^{x/y} \cdot 1/y - xe^{x/y} - e^{x/y} \cdot 1/y}{y^2}$$

$$= \frac{-xe^{x/y}}{y^2}$$

$$\therefore \frac{d}{dx} e^{ax} = ae^{ax}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

clearly equation (1) is Exact Differential Equation

$\therefore u + \int \phi_y dy = C \rightarrow$  (2) is a General Soln of eqn (1)

$$u = \int (y) M dx$$

$\phi_y =$  Terms of 'N' not containing 'x'

$$u = \int M dx = \int (1 + e^{x/y}) dx$$

$$= \int 1 dx + \int e^{x/y} dx$$

$$= x + \frac{e^{x/y}}{(1/y)}$$

$$u = x + ye^{x/y}$$

$$\phi_y = 0$$

from (2)

$$x + ye^{x/y} + \int 0 dy = C$$

$$\Rightarrow x + ye^{x/y} = C$$